

Consider the autonomous DE $\frac{dy}{dx} = f(y)$, where $f(y)$ is the function whose graph is shown on the right.

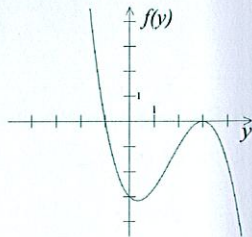
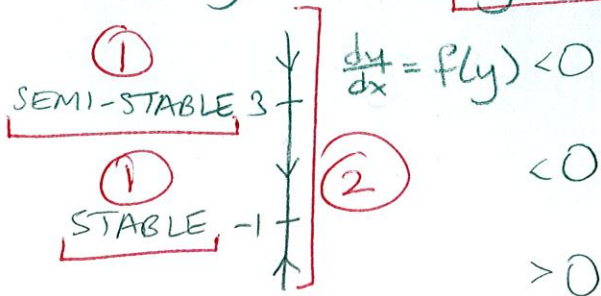
SCORE: ____ / 6 PTS

- [a] Find all equilibrium solutions of the DE and classify each as stable, unstable or semi-stable.

You must draw a phase portrait to get full credit.

$$f(y) = 0 \rightarrow y = -1, y = 3$$

EQUILIBRIUM
SOLUTIONS



- [b] If $y = m(x)$ is a solution of the DE such that $m(4) = 2$, what is $\lim_{x \rightarrow \infty} m(x)$?

$$\underline{-1} \quad \textcircled{1}$$

In certain population models, a group will go extinct if and only if its population is below a certain level (called the **SCORE: ____ / 4 PTS** survival threshold P_s). Write a differential equation for the population of a group which is going extinct, if the rate of change of its population is proportional to the difference between the threshold and the existing population. Justify your answer properly, but briefly.

NOTE: The signs of all constants should be stated clearly.

$$\boxed{\frac{dP}{dt} = k(P - P_s)} \quad \textcircled{2\frac{1}{2}}$$

GOING EXTINCT $\rightarrow$ $\textcircled{\frac{1}{2}}$ $\boxed{P < P_s}$ AND $\boxed{\frac{dP}{dt} < 0}$
 $\boxed{P - P_s < 0}$ $\textcircled{\frac{1}{2}}$

NOTE: THE BETTER ANSWER IS

$$\frac{dP}{dt} = -k(P_s - P) \text{ SINCE IT IS}$$

MORE "OBVIOUS" THAT $\frac{dP}{dt} < 0$

so $\boxed{k > 0}$
 $\textcircled{\frac{1}{2}}$

What does the Existence and Uniqueness Theorem tell you about possible solutions to the IVP

SCORE: ____ / 4 PTS

$$\frac{dy}{dx} = \frac{\sqrt[3]{y-2}}{x+6}, \quad y(8) = 2 ? \text{ Justify your answer properly, but briefly.}$$

$$f = (y-2)^{\frac{1}{3}}(x+6)^{-1}$$

$$f_y = \frac{1}{3}(y-2)^{-\frac{2}{3}}(x+6)^{-1} \text{ is } \textcircled{1} \\ = \frac{1}{3 \sqrt[3]{(y-2)^2} (x+6)}$$

$$\textcircled{\frac{1}{2}}$$

NOT CONTINUOUS IN AN OPEN REGION
AROUND $(8, 2)$ SINCE $f_y(8, 2)$ DNE

E + U TELLS US NOTHING

$$\textcircled{\frac{1}{2}}$$

Determine if $y = A\sqrt{x} + \frac{B}{x^2} + \frac{x^2}{4}$ is a family of solutions of the DE $4x^2y'' + 10xy' - 4y = 5x^2$.

SCORE: ____ / 6 PTS

State your conclusion clearly.

$$y = Ax^{\frac{1}{2}} + Bx^{-2} + \frac{1}{4}x^2$$

$$y' = \frac{1}{2}Ax^{-\frac{1}{2}} - 2Bx^{-3} + \frac{1}{2}x \quad (1)$$

$$y'' = -\frac{1}{4}Ax^{-\frac{3}{2}} + 6Bx^{-4} + \frac{1}{2} \quad (1)$$

$$4x^2y'' + 10xy' - 4y = \left[\begin{array}{l} -Ax^{\frac{1}{2}} + 24Bx^{-2} + 2x^2 \\ + 5Ax^{\frac{1}{2}} - 20Bx^{-2} + 5x^2 \\ - 4Ax^{\frac{1}{2}} - 4Bx^{-2} - x^2 \end{array} \right] \quad (1)$$

=

$$\frac{6x^2}{(2)}$$

$$\frac{NO}{(1)}$$

Consider the IVP $y' = 2y^2 - 3x$, $y(1) = -2$. Use Euler's method with $h = 0.2$ to estimate $y(1.4)$.

SCORE: ____ / 4 PTS

$$y(1.2) \approx y(1) + y'(1)(1.2 - 1)$$

$$= -2 + (2(-2)^2 - 3(1))(0.2) = -2 + 5(0.2) = -1$$

$$y(1.4) \approx y(1.2) + y'(1.2)(1.4 - 1.2)$$

$$\approx -1 + (2(-1)^2 - 3(1.2))(0.2) = -1 + (-1.6)(0.2) = -1.32$$